

D Summer Harder Problems

- (a) Solve the simultaneous equations $y = 2x^2 + x + 1$
 $y + 17 = 13x$. (3, 22)

(b) What does your answer say about the geometric relationship between the two in the xy -plane? Tangent
- Solve $\frac{5}{x-1} + \frac{9}{x+1} = 8$. $x = 2$ or $x = -\frac{1}{4}$
- Make x the subject of $\frac{ax-b}{cy+dx} = e$. $x = \frac{cey+b}{a-de}$
- Find the range of the function $f(x) = 7 + 4x - x^2$ if the domain is all the real numbers. [Hint: Sketch] $f(x) \leq 11$
- Given $f(x) = \frac{ax}{b-x}$ find
 - $f^{-1}(x)$. $f^{-1}(x) = \frac{bx}{a+x}$
 - $ff(x)$. $ff(x) = \frac{a^2x}{b^2-bx-ax}$
 - $fff(x)$. $fff(x) = \frac{a^3x}{b^3-b^2x-a^2x-abx}$
- Given $g(x) = \frac{3x-a}{7-kx}$ find
 - $g^{-1}(x)$. $g^{-1}(x) = \frac{7x+a}{3+kx}$
 - $gg(x)$. $gg(x) = \frac{9x-10a+akx}{49-10kx+ak}$
 - $ggg(x)$. $\frac{27x-79a+13akx-a^2k}{343-79kx+17ak-ak^2x}$
- The normal to $y = 3x^2 - x + 2$ has gradient 4. Find where on the curve the normal exists. $(\frac{1}{8}, \frac{123}{64})$
- Find the equation of the tangent to $y = x^2 + 2x - 1$ when $x = p$ in the form $ax + by + c = 0$. $(2p+2)x + y + 1 + 2p^2 = 0$
- Find the equation of the normal to $y = \frac{1}{2x^2}$ when $x = k$ in the form $ax + by + c = 0$. $k^3x - 2k^2y + 1 - k^4$
- The tangent to $y = x^2$ when $x = p$ crosses the x -axis at P and the y -axis at Q . If the origin is O , find the area of triangle OPQ . $\frac{p^3}{4}$
- Differentiate the following with respect to x :
 - $(x+2)(x-1)(x+3)$. $3x^2 + 8x + 1$
 - $(x+a)(x-a)(x+b)$. $3x^2 + 2bx - a^2$
 - $3\sqrt{x} - \frac{1}{2x}$. $\frac{3}{2\sqrt{x}} + \frac{1}{2x^2}$
 - $\frac{3x^3-7x^2+2x-5}{x^2}$. $3 - \frac{2}{x^2} + \frac{10}{x^3}$
 - $\frac{(\sqrt{x}+x^2)(x-3)}{\sqrt{x}}$. $1 + \frac{5}{2}x^{\frac{3}{2}} - \frac{9}{2}\sqrt{x}$
- Find the point of intersection of the tangent to $y = x^3 - 3x + 1$ when $x = 1$ and the normal when $x = -1$. (-1, -1)
- Find the point of intersection of the tangent to $y = (2x-1)(x+3)$ when $x = 1$ and the normal when $x = 2$. $(\frac{131}{59}, \frac{884}{59})$