D Michaelmas Differentiation III

Patrons are reminded that $\frac{dy}{dx}$ *is* the gradient. Also, if a curve passes through a point, then the x and y values of the point fit into the equation of the curve.

1. A curve is given by $y = ax^2 + 2x - 1$. When $x = 1$, $\frac{dy}{dx} = 12$. Find a. $\boxed{a = 5}$

2. A curve is given by $y = 8x^3 + ax + 1$. When $x = -1$, $\frac{dy}{dx} = 23$. Find a. $\boxed{a = -1}$

3. A curve is given by $y = ax^2 + ax + 1$. When $x = 4$, $\frac{dy}{dx} = 27$. Find a. $\boxed{a = 3}$

4. A curve is given by $y = ax^2 + bx + 4$. It passes through the point $(1, 5)$. At that point the curve has gradient 4. Find a and b. $\boxed{a = 3, b = -2}$

5. A curve is given by $y = mx^2 + 3x + n$. It passes through the point $(1, 1)$. At that point the curve has gradient 7. Find m and n. $\boxed{m = 2, n = -4}$

6. A curve is given by $y = ax^2 + ax + b$. It passes through the point $(1, 10)$. At that point the curve has gradient 9. Find a and b. $\boxed{a = 3, b = 4}$

7. A curve is given by $y = x^3 + ax + b$. It passes through the point $(2, 14)$. At that point the curve has gradient 15. Find a and b. $\boxed{a = 3, b = 0}$

8. A curve is given by $y = x^3 + ax^2 + bx + 2$. It passes through the point $(-1, 12)$. At that point the curve has gradient -13. Find a and b. $\boxed{a = 5, b = -6}$

9. A curve is given by $y = ax^4 + bx + 1$. It passes through the point $(2, 23)$. At that point the curve has gradient 35. Find a and b. $\boxed{a = 1, b = 3}$

10. A curve is given by $y = 2x^3 + ax$, where a is a constant. The value of $\frac{dy}{dx}$ when $x = 2$ is twice the value of $\frac{dy}{dx}$ when $x = -1$. Work out the value of a. $\boxed{a = 12}$

11. A curve is given by $y = x^2 + kx$, where k is a constant. The value of $\frac{dy}{dx}$ when $x = 6$ is three times the value of $\frac{dy}{dx}$ when $x = 0$. Work out the value of k. $\boxed{k = 6}$

12. A curve is given by $y = mx^2 + 4x + 3$, where m is a constant. The value of $\frac{dy}{dx}$ when $x = 8$ is three times the value of $\frac{dy}{dx}$ when $x = 2$. Work out the value of m. $\boxed{m = 2}$

13. A curve is given by $y = 4x^2 + ax$, where a is a constant. The value of $\frac{dy}{dx}$ when $x = 4$ is five times the value of $\frac{dy}{dx}$ when $x = 1$. Work out the value of a. $\boxed{a = -2}$

14. A curve is given by $y = 5x^3 + kx$, where k is a constant. The value of $\frac{dy}{dx}$ when $x = 2$ is seven times the value of $\frac{dy}{dx}$ when $x = 0$. Work out the value of k. $\boxed{k = 10}$

15. A curve is given by $y = 4\sqrt{x} + ax$, where a is a constant. The value of $\frac{dy}{dx}$ when $x = \frac{1}{16}$ is three times the value of $\frac{dy}{dx}$ when $x = 1$. Work out the value of a. $\boxed{a = 1}$