Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Tuesday 26 June 2012 – Morning

Time: 3 hours

Materials required for examination	Items included with question papers
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

1. The function f is given by

$$f(x) = x^2 - 2x + 6, \qquad x \geqslant 0$$

(a) Find the range of f.

(3)

The function g is given by

$$g(x) = 3 + \sqrt{(x + 4)}, \qquad x \geqslant 2$$

(b) Find $gf(x)$.

(2)

(c) Find the domain and range of gf.

(3)

(Total 8 marks)

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2. (a) Show that

$$\sin 3x = 3\sin x - 4\sin^3 x$$

(3)

Hence find

(b) $\int \cos x(6\sin x - 2\sin 3x)^{\frac{2}{3}} \, dx$

(3)

(c) $\int (3\sin 2x - 2\sin 3x \cos x)^{\frac{1}{3}} \, dx$

(4)

(Total 10 marks)

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3. The angle θ, $0 < \theta < \frac{\pi}{2}$, satisfies

$$\tan\theta \tan 2\theta = \sum_{r=0}^{\infty} 2\cos^r 2\theta$$

(a) Show that $\tan\theta = 3^p$, where p is a rational number to be found.

(8)

(b) Hence show that $\frac{\pi}{6} < \theta < \frac{\pi}{4}$

(2)

(Total 10 marks)

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4. $$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ -2 \\ 9 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix}$$

The points A, B and C with position vectors **a**, **b** and **c**, respectively, are 3 vertices of a cube.

(a) Find the volume of the cube.

(5)

The points P, Q and R are vertices of a second cube with $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 4 \\ \alpha \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$

and α a positive constant.

(b) Given that angle $QPR = 60°$, find the value of α.

(3)

(c) Find the length of a diagonal of the second cube.

(3)

(Total 11 marks)

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5. [In this question the values of a, x, and n are such that a and x are positive real numbers, with $a > 1$, $x \neq a$, $x \neq 1$ and n is an integer with $n > 1$]

Sam was confused about the rules of logarithms and thought that

$$\log_a x^n = (\log_a x)^n \qquad (1)$$

(a) Given that x satisfies statement (1) find x in terms of a and n.

(3)

Sam also thought that

$$\log_a x + \log_a x^2 + ... + \log_a x^n = \log_a x + (\log_a x)^2 + ... + (\log_a x)^n \qquad (2)$$

(b) For $n = 3$, x_1 and x_2 ($x_1 > x_2$) are the two values of x that satisfy statement (2).

(i) Find, in terms of a, an expression for x_1 and an expression for x_2.

(ii) Find the exact value of $\log_a\left(\frac{x_1}{x_2}\right)$.

(5)

(c) Show that if $\log_a x$ satisfies statement (2) then

$$2(\log_a x)^n - n(n+1)\log_a x + (n^2 + n - 2) = 0$$

(6)

(Total 14 marks)

6.

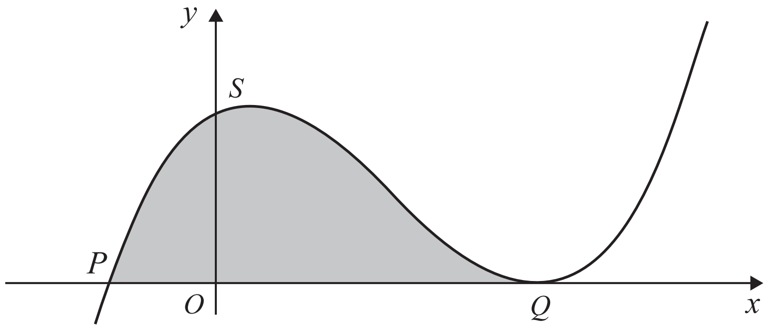

Figure 1

Figure 1 shows a sketch of the curve with equation $y = (x + a)(x - b)^2$, where a and b are positive constants. The curve cuts the x-axis at P and has a maximum point at S and a minimum point at Q.

(a) Write down the coordinates of P and Q in terms of a and b.

(2)

(b) Show that G, the area of the shaded region between the curve PSQ and the x-axis, is given by $G = \frac{(a+b)^4}{12}$.

(6)

The rectangle $PQRST$ has RST parallel to QP and both PT and QR are parallel to the y-axis.

(c) Show that $\frac{G}{\text{Area of } PQRST} = k$, where k is a constant independent of a and b and find the value of k.

(8)

(Total 16 marks)

7. [$\arccos x$ and $\arctan x$ are alternative notation for $\cos^{-1} x$ and $\tan^{-1} x$ respectively]

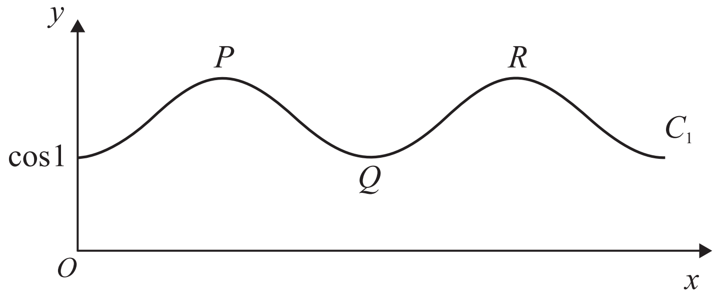

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \cos(\cos x)$, $0 \leqslant x < 2\pi$.

The curve has turning points at (0, cos1), P, Q and R as shown in Figure 2.

(a) Find the coordinates of the points P, Q and R.

(4)

The curve C_2 has equation $y = \sin(\cos x)$, $0 \leqslant x < 2\pi$. The curves C_1 and C_2 intersect at the points S and T.

(b) Copy Figure 2 and on this diagram sketch C_2 stating the coordinates of the minimum point on C_2 and the points where C_2 meets or crosses the coordinate axes.

(5)

The coordinates of S are (α, d) where $0 < \alpha < \pi$.

(c) Show that $\alpha = \arccos\left(\frac{\pi}{4}\right)$.

(2)

(d) Find the value of d in surd form and write down the coordinates of T.

(3)

The tangent to C_1 at the point S has gradient $\tan\beta$.

(e) Show that $\beta = \arctan\sqrt{\left(\frac{16-\pi^2}{32}\right)}$.

(5)

(f) Find, in terms of β, the obtuse angle between the tangent to C_1 at S and the tangent to C_2 at S.

(5)

(Total 24 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 100 MARKS

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