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Mark Scheme (Results)
Summer 2010

AEA

AEA Mathematics (9801)

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9801 Advanced Extension Award Mathematics
Mark Scheme

Q.	Scheme	Marks	Notes
1(a)	$3x+16=9+x+1+6\sqrt{x+1}$	M1	Initial squaring -both sides
	$3+x=3\sqrt{x+1}$ (o.e.)	A1	Correct collecting of terms
	$9+6x+x^2=9(x+1)$ <u>or</u> $y=\sqrt{x+1}\rightarrow$ 3TQ in y	M1	2nd squaring o.e.
	$x^2-3x=0$ <u>or</u> $(y-2)(y-1)=0$	A1	Both values
	<u>**x = 0 or 3**</u>	B1 (5)	(S+ for checking values)
(b)	$\frac{1}{2}\log_3 x=\log_3\sqrt{x}$	B1	For use of $n\log x$ rule
	$\log_3(x-7)-\log_3\sqrt{x}=\log_3\dfrac{x-7}{\sqrt{x}}$	M1	For reducing *xs* to a single log
	So $2x-14=3\sqrt{x}$ (o.e. all x terms on same line)	M1A1	M1 for getting out of logs A1 for correct equation
	$2\left(\sqrt{x}\right)^2-3\sqrt{x}-14=0$	M1	Attempt to solve suitable 3TQ in x or $\sqrt{x}$
	$\left(2\sqrt{x}-7\right)\left(\sqrt{x}+2\right)=0$		
	$\sqrt{x}=\dfrac{7}{2}$ or -2	A1	Either solution for $\sqrt{x}$ or x. Must be rational *a/b*
	<u>$x=\dfrac{49}{4}$</u>	A1 (7)	49/4 oe only (S+ for clear reason for rejecting x = 4)
		[12]	

Q.	Scheme	Marks	Notes
2(a)	$q = \frac{p}{2}(2a + (p-1)d)$ and $p = \frac{q}{2}(2a + (q-1)d)$	M1 A1	Attempt one sum formula Both correct expressions
	$2\left(\frac{q}{p} - \frac{p}{q}\right) = d\left(p - 1 - q + 1\right)$	dM1 A1	Eliminate *a*. Dep on 1st M1 Must use 2 indep. eqns Correct elimination of *a*
	$d = \frac{2(q^2 - p^2)}{pq(p-q)}$; $\quad \underline{d = \frac{-2(p+q)}{pq}}$	A1 (5)	Correct simplified *d* =
(b)	$2a = \frac{2q}{p} + \frac{(p-1)2(q+p)}{pq}$; $\quad a = \frac{q^2(q-1) - p^2(p-1)}{pq(q-p)}$	M1	Substitute for *d* in a correct sum formula i.e. eqn in *a* only
		dM1	Rearrange to *a* = . Dep M1
	$\frac{q^2 + qp + p^2 - p - q}{pq}$ or $\frac{q^2 + (p-1)(q+p)}{pq}$ or $\frac{p^2 + (q-1)(q+p)}{pq}$	A1 (3)	Correct single fraction with denom = *pq*
(c)	$S_{p+q} = \frac{p+q}{2}\left(\frac{2q}{p} + \frac{(p-1)2(q+p)}{pq} + \frac{-2(p+q)}{pq}(p+q-1)\right)$	M1	Attempt sum formula with *n* = (*p*+*q*) and ft their *a* and *d*
	$= \frac{p+q}{2}\left[\frac{2(q^2 + qp + p^2 - p - q)}{pq} - \frac{2(p+q-1)(p+q)}{pq}\right]$	M1	Attempt to simplify- denominator = *pq* or 2*pq*
	$\frac{p+q}{pq}\left[-pq\right]$ = - [$p + q$]	A1 (3) **[11]**	A1for -(*p*+*q*) (S+ for concise simplification/factorising)

<u>Marks for Style Clarity and Presentation (up to max of 7)</u>

<u>S1 or S2</u>

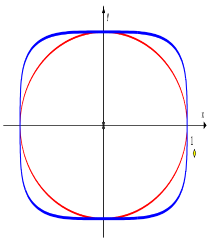
For a fully correct (or nearly fully correct) solution that is neat and succinct in question 1 to question 7

<u>T1</u>

For a good attempt at the whole paper. Progress in all questions.

Pick best 3 S1/S2 scores to form total.

Q.	Scheme	Marks	Notes
3(a)	$2x+2yy'+fy+fxy'=0$	M1	Correct attempt to diff'n y^2 or xy
		A1	All fully correct and = 0
	$\therefore y' = \dfrac{2x+fy}{-[2y+fx]}$	dM1	Isolate y' Dep on 1st M1
	At (α,β) gradient, $m = \dfrac{2\alpha+f\beta}{-[2\beta+f\alpha]}$ (o.e.)	A1 (4)	Sub α and β
(b)	m = 1 gives: $2\alpha+f\beta=-2\beta-f\alpha$	M1	Sub m = 1 and form linear equation in α and β.
	$\therefore(\alpha+\beta)(f+2)=0 \Rightarrow \alpha=-\beta \ \ (\text{or } f=-2)$ **(*)**	A1cso	(S+ for using $f \neq -2$)
	From curve: $\alpha^2+\alpha^2-f\alpha^2-g^2=0$ (o.e.)	M1	Sub $(\alpha=-\beta)$ into equation of curve
	$\therefore \alpha^2(2-f)=g^2 \Rightarrow \alpha^2=\dfrac{g^2}{2-f}$ and so $\alpha(\text{or } \beta)=\dfrac{\pm g}{\sqrt{2-f}}$ **(*)**	A1cso (4)	Simplify to answer. (S+ for considering $f<2$)
(c)	$(x-y)^2=g^2$ <u>or</u> $x-y=\pm g$	M1	Attempt to complete the square, allow <u>+</u> Or shows m = 1
	Line $y=x+g$ sketched	A1	Sketches should show y intercept or eq'n at least.
	Line $y=x-g$ sketched	A1 (3)	
		[11]	
4(a)	$\overrightarrow{AC}=\begin{pmatrix}-5\\10\\0\end{pmatrix}, \overrightarrow{AF}=\begin{pmatrix}0\\10\\20\end{pmatrix}$;	B1	Vectors AC or AF. Condone <u>+</u>
	$\lvert\overrightarrow{AC}\rvert=\sqrt{125}, \lvert\overrightarrow{AF}\rvert=\sqrt{500}$	B1	correct mods
	$\overrightarrow{AC}\bullet\overrightarrow{AF}=100 \Rightarrow \cos\angle CAF = \dfrac{100}{\sqrt{125}\sqrt{500}}, = \underline{\dfrac{2}{5}}$ or 0.4	M1	Complete method for <u>+</u> cos(CAF)
		A1 (4)	
(b)	$\overrightarrow{OX}=\begin{pmatrix}5\\0\\0\end{pmatrix}+t\begin{pmatrix}-5\\10\\0\end{pmatrix}=\begin{pmatrix}5-5t\\10t\\0\end{pmatrix}$ <u>or</u> $\begin{pmatrix}a\\10-2a\\0\end{pmatrix}$;	M1;	Attempt equation for AC or variable OX
	$\overrightarrow{FX}=\begin{pmatrix}-5t\\10t-10\\-20\end{pmatrix}$	<u>M1</u>	Attempt FX. Must be in terms of <u>one</u> unknown
	$\overrightarrow{FX}\bullet\overrightarrow{AC}=0 \Rightarrow 25t+100t-100+0=0$, [t = 0.8]	M1	Correct use of . to get linear eqn in t
		A1	t = 0.8 o.e.
	$\overrightarrow{OX}=\begin{pmatrix}1\\8\\0\end{pmatrix}$;	A1	Correct vector OX
	$\overrightarrow{FX}=\begin{pmatrix}-4\\-2\\-20\end{pmatrix}$ and $\lvert\overrightarrow{FX}\rvert=\sqrt{420}$	<u>M1</u>	Attempt <u>+</u> FX
		<u>A1</u> (7)	$\sqrt{420}$ o.e.
	[$\lvert\overrightarrow{FX}\rvert=\sqrt{420}$ earns <u>M1</u> <u>M1</u> <u>A1</u> ; $\overrightarrow{OX}$ earns M1M1A1A1]		
(c)	$l_1:(\mathbf{r}=)\lambda\begin{pmatrix}5\\5\\10\end{pmatrix}$ and $l_2:(\mathbf{r}=)\begin{pmatrix}5\\0\\0\end{pmatrix}+\mu\begin{pmatrix}-2.5\\10\\20\end{pmatrix}$	B1 B1	B1 for each vector equation
	Solving: $5\lambda=5-2.5\mu$ and $5\lambda=10\mu$ (o.e.)	M1	Clear attempt to solve leading to λ = or μ = Either
	$\lambda=0.8, \quad \mu=0.4$	A1	Accept position vector
	Intersection at the point <u>**(4, 4, 8)**</u>	A1 (5)	(S+ for clear attempt to check intersection)
		[16]	

Q.	Scheme	Marks	Notes
5(a)	$x = 1 + u^{-1} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u} = -\frac{1}{u^2}$	B1	Correct dx/du (o.e.)
	$\therefore I = \int \frac{1}{u^{-1}\sqrt{u^{-2}+2u^{-1}}}.\left(-\frac{1}{u^2}\right)\mathrm{d}u$	M1	Attempt to get I in u only
	$I = -\int \frac{\mathrm{d}u}{\sqrt{1+2u}}$ (o.e)	A1	Correct simplified expression in u only
	$= -(1+2u)^{\frac{1}{2}}\ (+c)$	M1 A1	Attempt to int' their I Correct integration
	Uses $u = \frac{1}{x-1}$ to give $I = -(1 + \frac{2}{x-1})^{\frac{1}{2}} + c$, $I = -\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}} + c$	M1 A1cso (7)	Sub back in xs Including $+ c$
(b)	$= -\left(\frac{\sec\beta+1}{\sec\beta-1}\right)^{\frac{1}{2}} + \left(\frac{\sec\alpha+1}{\sec\alpha-1}\right)^{\frac{1}{2}}$	M1	Use of part (a)
	$= -\left(\frac{1+\cos\beta}{1-\cos\beta}\right)^{\frac{1}{2}} + \left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)^{\frac{1}{2}}$	M1	Multiply by cos$x$
	$= -\left(\frac{2\cos^2(\frac{\beta}{2})}{2\sin^2(\frac{\beta}{2})}\right)^{\frac{1}{2}} + \left(\frac{2\cos^2(\frac{\alpha}{2})}{2\sin^2(\frac{\alpha}{2})}\right)^{\frac{1}{2}}$ ["2" is needed]	M1	Use of half angle formulae
	$= \cot\left(\frac{\alpha}{2}\right) - \cot\left(\frac{\beta}{2}\right)$ (*)	M1 A1cso (5) **[12]**	Correct removal of $\sqrt{\ }$.
6(a)	$A = x^2 + y^2 = x^2 + (1-x^4)^{\frac{1}{2}}$	B1	A as function of x only
	$\therefore \frac{\mathrm{d}A}{\mathrm{d}x} = 2x - (2x^3)(1-x^4)^{-\frac{1}{2}}$	M1	For some correct diff'n. More than just $2x$
	$\frac{\mathrm{d}A}{\mathrm{d}x} = 0,\quad x = 0$ or $x^2 = (1-x^4)^{\frac{1}{2}}$	A1 B1	For $x^2 = (1-x^4)^{\frac{1}{2}}$ For $x = 0$[$\Rightarrow$ by min = 1]
	i.e. $x^2 = y^2 \Rightarrow x = \pm y$; and $x^4 = y^4 = \frac{1}{2}$, so $x^2 + y^2 = \sqrt{2}$	M1; B1	M1 for reaching $y = \pm x$ B1 for max = $\sqrt{2}$
	So minimum is 1 [and maximum is $\sqrt{2}$]	B1 (7)	For min = 1
(b)		B1 B1	Circle, centre (0,0) r =1 Other curve
(c)	$x^2 + y^2 = \sqrt{2}$	B1 (3) **[10]**	(S+ for some explanation
ALT(a)	Let $x = r\cos\theta$ and $y = r\sin\theta$ then $r^4(\cos^4\theta + \sin^4\theta) = 1$	B1	
	$r^4 = \frac{1}{\cos^4\theta + \sin^4\theta} = \frac{1}{1 - \frac{1}{2}\sin^2 2\theta}$; So $1 < r^2 < 2$	M1A1; B1B1	
	Max value when $\theta = \frac{\pi}{4}$ so $x = y$	M1A1	
OR	$A^2 = (x^2+y^2)^2 = 1 + 2x^2y^2 = 1 + 2x^2\sqrt{(1-x^4)}$	1st B1	Then differentiate as before
OR	$A^2 - 1 = 2x^2y^2 \rightarrow (A^2-1)^2 = 4x^4(1-x^4)$; $= 4(\frac{1}{4} - (\frac{1}{2} - x^4)^2)$	B1:M1A1	By completing the square

Q.	Scheme	Marks	Notes
7(a)	$f(x)=[1+(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4})][1+(\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4})]$	M1	Use of sin($A\pm B$) etc
	$=[1+\frac{1}{\sqrt2}\cos x-\frac{1}{\sqrt2}\sin x)][1+\frac{1}{\sqrt2}\sin x+\frac{1}{\sqrt2}\cos x]$	B1	$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{1}{\sqrt2}$
	$=(1+\frac{1}{\sqrt2}\cos x)^2-(\frac{1}{\sqrt2}\sin x)^2$ **or** $=1+\frac{2}{\sqrt2}\cos x+\frac12\cos^2 x-\frac12\sin^2 x$ $=1+\frac{2}{\sqrt2}\cos x+\frac12\cos^2 x-\frac12(1-\cos^2 x)$	M1	Multiply out and remove sin$x$cos$x$ terms
	So f(x) $=\frac12+\frac{2}{\sqrt2}\cos x+\cos^2 x$ $=(\frac{1}{\sqrt2}+\cos x)^2$ **(*)**	M1 A1cso (5)	Eqn in cos$x$ only
(b)	Range: $0\le f(x)\le(\frac{1}{\sqrt2}+1)^2$ or equivalent e.g. $\frac32+\frac{2}{\sqrt2}$	M1 A1 (2)	M1 f$\ge$0 <u>or</u> f$\le(\frac{1}{\sqrt2}+1)^2$ A1 both [M1A0 for <]
(c)	$\cos x=1$ gives maxima at $(0,\frac32+\sqrt2)$ and at $(2\pi,\frac32+\sqrt2)$	B1 B1ft	If y co-ord is wrong allow 2nd B1ft
	Minima when $(\frac{1}{\sqrt2}+\cos x)=0\Rightarrow\cos x=-\frac{1}{\sqrt2}$ so at $x=\frac{3\pi}{4}$ or $\frac{5\pi}{4}$	M1 A1	M1 for y = 0 at cos$x$ = A1 for x co-ords
	$f'(x)=-2\sin x(\frac{1}{\sqrt2}+\cos x)=0$ at $x=\pi$, so at $(\pi,\frac32-\sqrt2)$ there is a (local) maximum	M1 A1 (6)	For f'(x)=0 and $x=\pi$ A1for max point
(d)	y = 2 meets y = f(x) so $(\frac{1}{\sqrt2}+\cos x)^2=2\Rightarrow\cos x=\frac{\sqrt2}{2}$ $\therefore x=\frac{\pi}{4}$ or $\frac{7\pi}{4}$	M1 A1	Form and solve correct eqn Both
	Area = $\int(2-f(x))\,dx$ [or correct rect - integral o.e.]	M1	Correct strategy
	$=\int\left(1-\sqrt2\cos x-\frac12\cos 2x\right)dx$	M1	All terms of integral in suitable form
	$=\left[x-\sqrt2\sin x-\frac14\sin 2x\right]$	dM1A1	M1 for some correct int' Dep on previous M A1 for all correct
	$=\left(\frac{7\pi}{4}+\sqrt2\times\frac{1}{\sqrt2}+\frac14\times1\right)-\left(\frac{\pi}{4}-\sqrt2\times\frac{1}{\sqrt2}-\frac14\right)$	dM1	Use of their correct limits. Dep on 1st M1
	$=\underline{\frac{3\pi}{2}+\frac52}$	A1 (8) **[21]**	NB Rectangle $=3\pi$
ALT	(a) $f(x)=1+\sqrt2\cos(x+\frac{\pi}{4}-\frac{\pi}{4})+\frac12\sin(2x+\frac{\pi}{2})$ $=1+\sqrt2\cos x+\frac12\cos 2x$ $=1+\sqrt2\cos x-\frac12+\cos^2 x$	1st M1B1 2nd M1 3rd M1	Remove sin($2x+\frac{\pi}{2}$) Then as in scheme
ALT	(d) $\int(\frac{1}{\sqrt2}+\cos x)^2 dx=\int\frac12+\sqrt2\cos x+\frac12+\frac12\cos 2x\,dx$ $=\frac12 x+\sqrt2\sin x+\frac14\sin 2x+\frac12 x$	3rd M1 4th M1 2nd A1	All terms in form to int' Will score 2nd M1 when they try to subtract from area of rectangle

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