Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics

Advanced Extension Awards AEA (9801)

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AEA June 2008

	NOTES	MARKS
1. $a=200,\ d=-\frac{5}{2}$ $\quad u_n=0 \Rightarrow 200-\frac{5}{2}(n-1)=0$	Identify a, d and set $u_n=0$	M1
$\Rightarrow \quad n=81$		A1
[ALT $S_n=\frac{n}{2}\left(400-\frac{5}{2}(n-1)\right)$  Max at 80.5	S_n and attempt max	M1 A1]
Maximum sum when $n=80$ or 81	Use of S_n with their $n=80$ or their $n=81$	M1 A1
$S_{80}=40\left[400-\frac{5}{2}\times 79\right]$		
$=20[800-395]$		A1 (5)
$=\underline{\underline{8100}}$		
2. (a) $\frac{dy}{dx}=2 \Rightarrow 2(x+1)(x+2)=xy$	Sub $\frac{dy}{dx}=2$	M1
$\Rightarrow 2(x^2+3x+2)=x(2x+5)$	Sub y for 2x+5 and attempt to solve	M1
$y=2x+5 \Rightarrow$ $x=-4$, $y=-3$ [or P is $(-4,-3)$]		A1 A1 (4)
(b) $\int\frac{1}{y}\,dy=\int\frac{x}{(x+1)(x+2)}\,dx$	Separation attempt	M1
$=\int\left(\frac{2}{x+2}-\frac{1}{x+1}\right)dx$	Attempt partial fractions	M1
$\Rightarrow \ln\lvert y\rvert = 2\ln\lvert x+2\rvert-\ln\lvert x+1\rvert\ (+c)$	Some correct ln integral of x function	M1 A1
$\ln y=\ln\left[\frac{A(x+2)^2}{(x+1)}\right]$ or $\ln\left[\frac{(x+2)^2}{(x+1)}\right]+c$	Use of log rules $\to \ln[g(x)]$ (condone $A=1$ or $c=0$)	M1
$y=\frac{A(x+2)^2}{(x+1)}$	Getting out of logs (must have 'A' or equiv)	M1
Using $P(-4,-3) \Rightarrow -3=\frac{A(-2)^2}{(-3)}$ $\quad$ ($\int$ their P)	Use P to form eqn in A or C	M1
$y=\frac{9(x+2)^2}{4(x+1)}$		A1 (8)
	Max S1 only for 11 or 12/12	(12)

S1 or S2 For a fully correct (or nearly so) and neat or succinct solution to Qu2 - Qu7. Count best 3 questions.

T1 For a good attempt at the whole paper (all questions)

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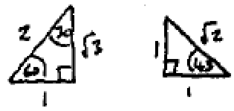
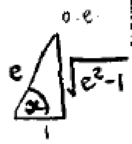
3 (a) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$; $t=\tan 15$ 	Use of known $\triangle$s...	B1
$\tan 30 = \dfrac{1}{\sqrt3} \Rightarrow \dfrac{1}{\sqrt3} = \dfrac{2t}{1-t^2}$	Use ~~tan2θ~~ $\rightarrow$ equation in t	M1
$t^2 + 2\sqrt3 t - 1 = 0 \quad \Rightarrow t = \dfrac{-2\sqrt3 \pm\sqrt{12+4}}{2}$	Attempt to solve $\rightarrow t=$	M1
$t=\tan 15 = \underline{\underline{2-\sqrt3}}$ (*)	[S for considering ±]	A1 cso (4)
(b) $\left(\dfrac{\sin\theta}{2} + \dfrac{\sqrt3}{2}\cos\theta\right)\left(\dfrac{\sin\theta}{2} - \dfrac{\sqrt3}{2}\cos\theta\right) = \cos^2\theta(1-\sqrt3)$	Use of $\sin(A\pm B)$	M1
$\dfrac{\sin^2\theta}{4} - \dfrac{3}{4}\cos^2\theta = \cos^2\theta - \sqrt3\cos^2\theta$	Equation in s^2 and c^2 or c^2 and $\cos 2\theta$	M1
$\dfrac{\sin^2\theta}{4} = \dfrac{\cos^2\theta}{4}(7-4\sqrt3)$	Attempt $\cos^2\theta$, ~~$\tan^2\theta$~~ or $\cos 2\theta$ or $\sin^2\theta$	M1
$\cos^2\theta = \dfrac{1}{4(2-\sqrt3)}$ or $\dfrac{2+\sqrt3}{4}$ or $\tan^2\theta = 7-4\sqrt3$ or $\cos 2\theta = \dfrac{2\sqrt3-3}{4-2\sqrt3}$		A1
$\tan^2\theta = (2-\sqrt3)^2$	$(2-\sqrt3)^2 = 7-4\sqrt3$	M1
$\tan\theta = \pm(2-\sqrt3)$ or $\cos 2\theta = \dfrac{\sqrt3}{2}$		A1
$\tan\theta = 2-\sqrt3 \Rightarrow \underline{\underline{\theta = 15, 195}}$; $\tan\theta = -(2-\sqrt3) \Rightarrow \underline{\underline{\theta = 165, 345}}$		A1; A1 (8) 12
4. (a) $\dfrac{dy}{dx} = -\sin x \ln(\sec x) + \cos x . \tan x$	Use of product rule	M1 A1
$y'=0 \Rightarrow 0 = \sin x\,(1-\ln(\sec x))$	Take out $\sin x$ factor	M1
[$\sin x = 0 \Rightarrow x = 0 \quad \therefore$ Min at origin]	[S marks]	
$\ln\sec x = 1 \Rightarrow \sec x = e$; $\therefore \underline{B \,(\arccos\frac1e, \frac1e)}$		A1 ; A1 (5)
(2) [Rectangle − S]	For strategy	M1
(b)  $I = \int \cos x \ln(\sec x)\,dx = \sin x \ln\sec x - \int \sin x\tan x\,dx$	Attempt parts	M1 A1
$I = \sin x\ln\sec x - \int \dfrac{\sin^2 x}{\cos x}\,dx = \sin x\ln\sec x - \int(\sec x - \cos x)\,dx$	Put $\sin x\tan x$ into integrable form	M1
$I = \sin x\ln\sec x - \ln\lvert\sec x + \tan x\rvert + \sin x$	correct integration	A1 A1
$S = [I]_0^{\arccos\frac1e}$ 	Attempt correct limits and $\sin x$ and $\cos x$ in terms of e, $\sqrt{e^2-1}$ etc.	M1 Must have complete integral
$S = \dfrac{\sqrt{e^2-1}}{e} - \ln\left[e+\sqrt{e^2-1}\right] + \dfrac{\sqrt{e^2-1}}{e}$		
Area $= 2\left[\frac1e\arccos\frac1e - S\right] = \underline{\underline{\dfrac{2}{e}\arccos\dfrac1e + 2\ln\left(e+\sqrt{e^2-1}\right) - \dfrac{4\sqrt{e^2-1}}{e}}}$ (*)		A1 cso. (8) 13

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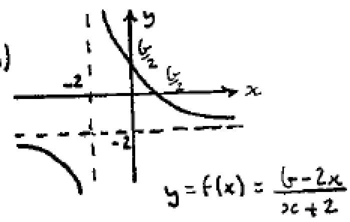
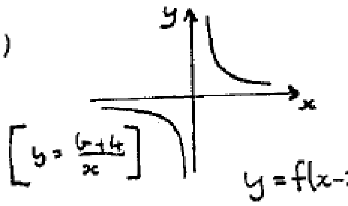
5. (i) $(\log_3 p)^2 = \log_3(p^2) \Rightarrow (\log_3 p)^2 = 2\log_3 p$	Use $n\log x$ rule	M1
$\Rightarrow \log_3 p(\log_3 p - 2) = 0 \Rightarrow \log_3 p = 0 \therefore p = 1$		A1
$\log_3 p = 2 \therefore p = 9$		A1
or		
$\log_3(p+q) = \log_3 p + \log_3 q \Rightarrow \log_3(p+q) = \log_3(pq)$	Use of $\log x + \log y$ rule	M1
		A1
$\Rightarrow p + q = pq$ OR $9 + q = 9q$		
$\therefore q = \frac{p}{p-1}$ $(p \neq 1)$	Making q the subject [S for $p \neq 1$]	M1
		A1 (7)
$\underline{p = 9} \Rightarrow \underline{q = \frac{9}{8}}$		
(ii) $\log_3 9 = 2$ o.e.	Seen anywhere	B1
$\log_3\left[\frac{3x^3 - 23x^2 + 40x}{(3x-8)^2}\right] = 1$	Use of log rules to form a single log out of logs	M1 M1
$\frac{3x^3 - 23x^2 + 40x}{(3x-8)^2} = 3$	For reducing cubic equation to quadratic [$x \neq 8/3$ for S mark]	M1
$\frac{x(\cancel{3x-8})(x-5)}{(3x-8)^{\cancel{2}}} = 3$ $\quad 3x^2 - 44x + 96 = 0$		
$x^2 - 14x + 24 = 0$	3TQ (accept 3TQ)	A1
$x^2 - 5x = 9x - 24 \Rightarrow$		
$(x-12)(x-2) = 0$; $\Rightarrow x = 2$ or $\underline{12}$	(ignore $x = 2$ and $8/3$)	M1 ; A1 (7)
($x = 8/3$ listed here loses final A1)	[S marks for connection $2, 8/3$]	14
6. (a) $yx + 2y = ax + b \Rightarrow x = \frac{b-2y}{y-a} \therefore \underline{\underline{f^{-1}(x) = \frac{b-2x}{x-a}}}$	Make x the subject	M1, A1 (2)
(b) $ff(x) = x \Rightarrow f^{-1}(x) = f(x)$; $\therefore \underline{a = -2}$	$f^{-1} = f$	M1; A1 (2)
(c) (i)  $y = f(x) = \frac{b-2x}{x+2}$	(i) shape	B1 (no overlap)
	$x = -2$, $y = -2$	B1
	$(\frac{b}{2}, 0)$, $(0, \frac{b}{2})$	B1 (3)
(ii)  $\left[y = \frac{b+4}{x}\right]$ $y = f(x-2) + 2$	(ii) $\rightarrow +2$	M1
	$\uparrow +2$	M1
	both branches	A1 (3)
(d) Normal at P′ on $y = f(x-2) + 2$ is: $y = 4(x-2) - 39 + 2$	Use transformations on normal	M1
$y = 4x - 45$		A1
$\therefore$ curve is symmetric about $y = \pm x$, normal at Q′ will be $y = 4x + 45$ [symmetry is $x \to -x$ and $y \to -y$]	Use symmetry on Q′	M1
Reversing process		
Normal at Q on $y = f(x)$ is : $y = 4(x+2) + 45 - 2$	Use $f(x+2) - 2$	M1
$\therefore \underline{y = 4x + 51}$ or $k = 51$		A1 (5)
		15
[N.B. P′ = (12, 3) P = (10, 1) ; $b = 32$; Q = (−14, −5)] M1A1 M1 $y - -5 = 4(x - -14) \to k = 51$	M1 A1	

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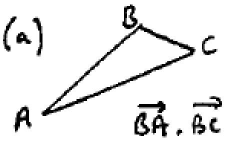
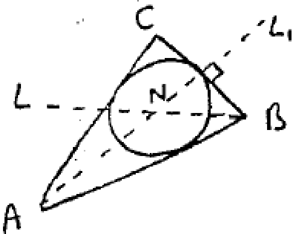
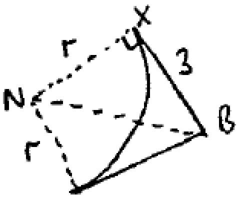
7. (a)  $\vec{BA} = \begin{pmatrix}-4\\1\\-8\end{pmatrix}$ $\quad \vec{BC} = \begin{pmatrix}4\\2\\-4\end{pmatrix}$	Attempt $\vec{BA}$ and $\vec{BC}$	M1
$\vec{BA}.\vec{BC} = -16+2+32 = 18$	Attempt $\vec{BA}.\vec{BC}$	M1
$\lvert\vec{BA}\rvert = \sqrt{4^2+1^2+8^2} = 9$, $\lvert\vec{BC}\rvert = \sqrt{4^2+2^2+4^2} = 6$	Attempt $\lvert\vec{BA}\rvert$ or $\lvert\vec{BC}\rvert$	M1
$\therefore \cos\theta = \frac{18}{9\times 6} = \frac{1}{3}$		A1 (4)
(b)  Using Rhombus idea, $\vec{BX} = \vec{BC} + \frac{2}{3}\vec{BA}$ o.e.	eg $3\vec{BC} + 2\vec{BA}$ Any correct ratio	M1, A1
$= \lambda\begin{pmatrix}4\\8\\-28\end{pmatrix}$ or $\mu\begin{pmatrix}1\\2\\-7\end{pmatrix}$		A1
Through B $\therefore$ $\underline{r} = \begin{pmatrix}1\\0\\-1\end{pmatrix} + t\begin{pmatrix}1\\2\\-7\end{pmatrix}$ (*)		A1 cso (4)
(c) $\vec{AC} = \begin{pmatrix}8\\1\\4\end{pmatrix}$ $\quad \lvert\vec{AC}\rvert = \sqrt{8^2+1^2+4^2} = 9 = \lvert\vec{BA}\rvert$	Attempt $\vec{AC}$ and $\lvert\vec{AC}\rvert$ (must say $= \lvert\vec{BA}\rvert$ for A1)	M1 A1 cso (2)
		
(d) $\because$ ABC is Isos L_1 has direction $\frac{1}{2}(\vec{AB}+\vec{AC}) = \begin{pmatrix}6\\0\\6\end{pmatrix}$		B1
$\therefore L_1$ has equation $(\underline{r} =) \begin{pmatrix}-3\\1\\-9\end{pmatrix} + u\begin{pmatrix}1\\0\\1\end{pmatrix}$	Find equation of L_1	M1
Centre of S is intersection of L_1 and L	Strategy	M1
Solving: $\left.\begin{matrix}1+t = -3+u\\ 2t = 1\end{matrix}\right\} \Rightarrow t = \frac{1}{2}$, $u = 9/2$	Attempt to solve $t =$ or $u =$	M1 A1
$[\,-1-7t = -9+u$ Check: LHS $= -\frac{9}{2}$, RHS $= -\frac{9}{2}\,]$		
$\therefore$ Centre has position vector $\vec{ON} = \begin{pmatrix}3/2\\1\\-9/2\end{pmatrix}$	(-1 eoo)	A2/1/0 (7)
(e)  Let X = mid-point of BC $\therefore$ BX = 3 ($\because$ Isos)	BX = 3	B1
$\vec{BN} = \begin{pmatrix}1/2\\1\\-7/2\end{pmatrix}$ $\quad \therefore \vec{BN} = \frac{1}{2}\sqrt{54}$	Attempt $\vec{BN}$ & $\lvert\vec{BN}\rvert$	M1 A1
$r^2 = BN^2 - 3^2$ $\quad \therefore r^2 = \frac{54}{4} - 9$. $\quad \therefore r = \frac{\sqrt{18}}{2}$ or $\frac{3}{2}\sqrt{2}$	Full method for r	M1 A1 (5)
		22